ECE 536 – Spring 2022

Homework #2 – Solutions

Problem 1)

(A) ISW BOUND STATES From the infinite square well model, we can express the energy in terms of the effective mass, the well width, and the energy level n. Rearranging for n and setting the maximum energy level to be the band edge height, we can see

$$E_n = \frac{n^2\hbar^2\pi^2}{2m^*L^2} \rightarrow n = \left\lfloor \sqrt{\frac{2m^*L^2}{\pi^2\hbar^2}\Delta E_c} \right\rfloor \tag{1.1}$$

Thus, we see there is only one bound state in the conduction band. If we replace the effective masses and potential height with their respective valence band values, we see there are 6 heavy hole states and 2 light hole states.

(B) ISW TRANSITION ENERGIES Now we can just use the formula above for the infinite square well energies

$$\begin{aligned} C_1 &= E_c + \frac{\hbar^2\pi^2}{2m^*L^2} = E_c + 91.7meV \\ HH_1 &= E_v - \frac{\hbar^2\pi^2}{2m^*L^2} = E_v - 7.6meV \\ LH_1 &= E_v - \frac{\hbar^2\pi^2}{2m^*L^2} = E_v - 76.0meV \end{aligned} \tag{1.2}$$

The transition energies are then just a subtraction (remembering that $E_c - E_v = E_g$). Thus the C_1 to HH_1 transition is 0.849 eV and the C_1 to LH_1 is 0.918 eV.

(C) ISW VS. FSW The actual transition energies would most likely be lower because of the infinite barrier approximation. When the finite barrier model is used (a better approximation that the one above), the energy states will leak into the barrier, increasing the effective width of the well compared to the infinite barrier model and thus reducing the energy of each state.

(D) FSW TRANSITION ENERGIES Using the finite barrier model described on p. 87 of the text, we can write

$$\begin{aligned} &\alpha'\tfrac{L}{2} = \sqrt{\tfrac{m_b}{m_w}}\,k\tfrac{L}{2}\tan k\tfrac{L}{2} && : even \\ &\alpha'\tfrac{L}{2} = -\sqrt{\tfrac{m_b}{m_w}}\,k\tfrac{L}{2}\cot k\tfrac{L}{2} && : odd \\ &\left(k\tfrac{L}{2}\right)^2 + \left(\alpha'\tfrac{L}{2}\right)^2 = \tfrac{2m_wV_0}{\hbar^2}\left(\tfrac{L}{2}\right)^2 \end{aligned}$$

The plots of these functions are shown in Fig. 1.1 for the conduction electron case. Similar plots can be made for both the heavy hole and light hole cases by simply adjusting for the effective mass and the potential height, given by the conduction and valence band offsets. After extracting the intersection points, we find that the energies are $E_{C1} = 39.1meV$, $E_{HH1} = 6.2meV$, $E_{LH1} = 36.2meV$, which are indeed lower than the infinite barrier model results above. Also note that there are more bound states than the infinite model allows for, since lower energy allows for more states to fit below the barrier height. The generating code is attached.

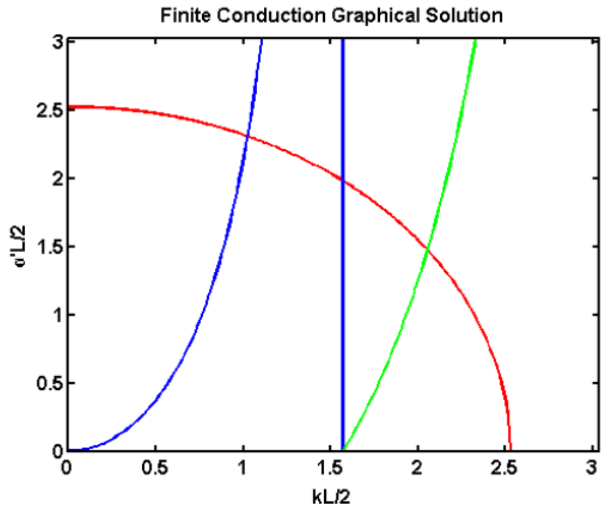

Figure 1.1: Graphical solution for the conduction band bound states using the finite barrier method.

Problem 2)

Auger Recombination in k-space

The k-Space Auger diagram from p. 75 of the text is given in Fig. 2.1. In the CHCC process, an electron from the conduction band recombines with a hole in the heavy hole band (1→1'), and the excess energy is transferred to an electron in the conduction which is excited further (2→2'). In The CHSH process, an electron in the conduction band recombines with a hole in the heavy hole band (1→1') and a hole is excited from the heavy hole band to the spin orbit band (2→2'). In the CHLH process, a conduction band electron recombines with a hole in the heavy hold band (1→1'), and the excess energy excites a heavy hole into the light hole band (2→2'). In the CHHH, the conduction band electron and the heavy hole recombine (1→1'), exciting a heavy hole to a higher energy in the heavy hole band (2→2').

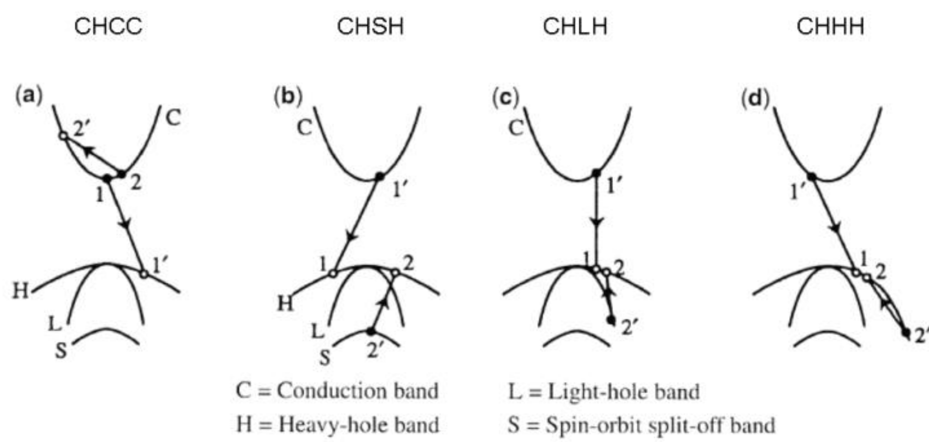

Figure 2.1: Auger recombination processes in k-Space, as given on p. 75 of the text.

Problem 3)

(A) DERIVATION To start with, we use the summation notation for the carrier density n and separate out the quantized direction so

$$n = \frac{2}{V}\sum_{\vec{k}} f(E) \rightarrow \sum_{m} \frac{2}{V}\sum_{k_x}\sum_{k_y} f(E) \tag{3.1}$$

Now we will follow the same process as before with the three-dimensional DOS where we cast k in terms of E. Starting from the parabolic band relation and accounting for quantization,

$$E = E_{c,m} + \frac{\hbar^2}{2m_e^*}k_{t,m}^2 \tag{3.2}$$

where m is the index of the quantized state in the $z-$direction, we see

$$k_{t,m} = \begin{matrix} \left(\frac{2m_e^*}{\hbar^2}\right)\left(E - E_{c,m}\right) & : E \geq E_{c,m} \\ 0 & : E < E_{c,m} \end{matrix}$$

Therefore, by finding $dk_{t,m}^2$, we can easily transform the summations above into integrals with respect to k and carry them out to find the two-dimensional DOS. The transform goes as follows:

$$\begin{aligned}\sum_m \frac{2}{V}\sum_{k_x}\sum_{k_y} f(E) \rightarrow \sum_m \iint \frac{1}{2\pi^2 L_z} d^2 k_{t,m} f(E) \\ = \sum_m \int_0^\infty \frac{1}{\pi L_z} k_{t,m} dk_{t,m} f(E) \\ = \sum_m \int_0^\infty \frac{m_e^*}{\pi L_z \hbar^2} H\left(E - E_{c,m}\right) f(E) dE \\ = \int_0^\infty \frac{m_e^*}{\pi L_z \hbar^2} \sum_m H\left(E - E_{c,m}\right) f(E) dE \\ \equiv \int_0^\infty \rho_{2D}(E) f(E) dE\end{aligned} \tag{3.3}$$

Then, by definition, $\rho_{2D} = \frac{m_e^*}{\pi L_z \hbar^2} \sum_m H\left(E - E_{c,m}\right)$.

(B) QUASI-FERMI LEVELS The first part of this section is simply brute force math. Write out the definition of the carrier density n in terms of the 2-D DOS derived above and the Fermi-Dirac distribution, still assuming an arbitrary number of occupied bands (which equates to leaving the infinite sum over the occupied states m)

$$\begin{aligned} n &= \int \rho(E) f(E) dE \\ &= \sum_m \int_0^\infty \frac{m_e^*}{\pi L_z \hbar^2} H\left(E - E_{c,m}\right) \frac{1}{1 + e^{(E-F_n)/k_B T}} dE \\ &= \sum_m \int_{E_{c,m}}^\infty \frac{m_e^*}{\pi L_z \hbar^2} \frac{1}{1 + e^{(E-F_n)/k_B T}} dE \\ &= \sum_m \int_{E_{c,m}}^\infty \frac{m_e^*}{\pi L_z \hbar^2} \frac{e^{(F_n - E)/k_B T}}{1 + e^{(F_n - E)/k_B T}} dE \end{aligned} \tag{3.4}$$

At this point, its best to make the substitution $u = \exp(F_n - E)/k_B T$. Ignoring the limits for now, we are left with

$$n = -k_B T \sum_m \frac{m_e^*}{\pi L_z \hbar^2} \int \frac{du}{1 + u} \tag{3.5}$$

This integral is easily performed by making the second substitution that $v = 1 + u$, leaving us with the final result

$$n = k_B T \frac{m_e^*}{\pi L_z \hbar^2} \sum_m \ln\left(1 + e^{(F_n - E_{c,m})/k_B T}\right) \tag{3.6}$$

which can be solved uniquely for a given carrier concentration n.

When the carrier density n increases, the quasi-Fermi level F_n will also increase. This can be observed mathematically from the integral equation, given that the density of states does no depend on the position of F_n. From a physical perspective, this means as n increases, more carriers will occupy the higher energy levels.

(C) ANALYTICAL QUASI-FERMI LEVELS If we assume a finite temperature, no, we cannot find a closed-form expression for $F_n(n)$, but we can express it as in part (b) to calculate. If the temperature is 0K, the Fermi-Dirac function becomes a step-function. In this case, we find the number of sub bands occupied and then rearrange the answer in part (b) to explicitly write $F_n(N)$.

When there is only a single sub-band occupied, we can proceed as in part (b). For brevity, I will not redo the integral.

$$\begin{aligned} n &= \int_{E_c}^{\infty} \frac{m_e^*}{\pi L_z \hbar^2} \frac{1}{1+e^{(E-F_n)/k_B T}} dE \\ &= k_B T \frac{m_e^*}{\pi L_z \hbar^2} \ln\left(1+e^{(F_n - E_c/k_B T}\right) \end{aligned} \tag{3.7}$$

Now simply rearrange to solve for the Fermi-level and see

$$F_n = E_c + k_B T \ln\left[\exp\left(\frac{n}{n_c}\right) - 1\right] \tag{3.8}$$

where $n_c \equiv k_B T \left(m_e^* / \pi L z \hbar^2\right)$.